A Brief Primer on Ideal Point Estimates

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Really took off with growth of game theory and EITM.
Claim: ideal points = player preferences.

Widely used in American, Comparative and International Relations: Measure legislator/judicial/voter/party/country preferences, gridlock intervals, uncovered sets, polarization, heterogeneity, representation, party pressure.

Outline:

- Statistical Models
- Computational Considerations
- Reasons for Concern

Claim 1:

The maps [of ideal points] are useless unless the user understands both the spatial theory that the computer program embodies and the politics of the legislature that produced the roll calls (Poole, 2005).

Claim 2:

Legislative voting is one of the most intensively studied areas of quantitative political science. Many have assumed that the basic answers were discovered long ago. Others will interpret the recent research as filling in 'everything they always wanted to know about legislative voting' and (often) didn't care to ask. But the challenge of the field remains (Weisberg, 1977).

Statistical Model

$$U_{it}\left(\theta_{y(t)}\right) = f(x_i, \theta_{y(t)}) + \zeta_{it}$$
$$U_{it}\left(\theta_{n(t)}\right) = f(x_i, \theta_{n(t)}) + \nu_{it}$$

Legislator i votes Yea if $f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)}) > \nu_{it} - \zeta_{it}$

$$\Pr(y_{it} = 1) = \Pr(\nu_{it} - \zeta_{it} < f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)}))$$

Ideal point estimators vary in assumptions about deterministic parts (f($x_i, \theta_{y(t)}$),f($x_i, \theta_{n(t)}$)) and stochastic parts (ζ_{it}, ν_{it}) of legislator i's utility function.

NOMINATE & Quadratic

Quadratic:

$$
\begin{aligned}
f(x_i, \theta_{y(t)}) &= -\left(x_i - \theta_{y(t)}\right)^2 \\
f(x_i, \theta_{n(t)}) &= -\left(x_i - \theta_{n(t)}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)}) &= -\left(x_i - \theta_{y(t)}\right)^2 + \left(x_i - \theta_{n(t)}\right)^2 \\
&= \beta_t x_i - \alpha_t
\end{aligned}
$$

Where: $\alpha_t = \theta^2_{n(t)} - \theta^2_{y(t)}$ and $\beta_t = 2\left(\theta_{y(t)} - \theta_{n(t)}\right)$.

NOMINATE & Quadratic

Quadratic:

$$\begin{aligned} f(x_i, \theta_{y(t)}) &= -\left(x_i - \theta_{y(t)}\right)^2 \\ f(x_i, \theta_{n(t)}) &= -\left(x_i - \theta_{n(t)}\right)^2 \end{aligned}$$

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NOMINATE:

$$\begin{aligned} f(x_i, \theta_{y(t)}) &= \beta \exp\left(-\frac{1}{2}\left(x_i - \theta_{y(t)}\right)^2\right) \\ f(x_i, \theta_{n(t)}) &= \beta \exp\left(-\frac{1}{2}\left(x_i - \theta_{n(t)}\right)^2\right) \end{aligned}$$

$$\begin{aligned} f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)}) = \ & \beta \left[\exp\left(-\frac{1}{2}\left(x_i - \theta_{y(t)}\right)^2\right) \right. \\ & \left. - \exp\left(-\frac{1}{2}\left(x_i - \theta_{n(t)}\right)^2\right)\right] \end{aligned}$$

Stochastic Elements

Assume ζ_{it} and ν_{it} are: uncorrelated and drawn from a symmetric, unimodal distribution. Suppose $\nu_{it} - \zeta_{it} \sim N(0, \sigma^2)$:

$$f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)}) \sim N(\nu_{it} - \zeta_{it}, \sigma^2)$$

Therefore:

$$\begin{aligned} \Pr(y_{it} = 1) = & \quad \Pr\left(\nu_{it} - \zeta_{it} < f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)}))\right) \\ = & \quad \Phi\left(\sigma^{-1}(f(x_i, \theta_{y(t)}) - f(x_i, \theta_{n(t)})))\right) \end{aligned}$$

Implying:

$$\Pr_N(y_{it} = 1) = \quad \Phi\left[\beta\left[\exp\left(-\tfrac{1}{2}\omega\left(x_i - \theta_{y(t)}\right)^2\right) - \exp\left(-\tfrac{1}{2}\omega\left(x_i - \theta_{n(t)}\right)^2\right)\right]\right]$$

$$\Pr_Q(y_{it} = 1) = \quad \Phi\left[\sigma^{-1}(\beta_t x_i - \alpha_t)\right]$$

Likelihood Functions

Quadratic Model:

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) = & \quad \Pi_{i=1}^{L}\Pi_{t=1}^{T}\Pr_Q(y_{it}=1)^{y_{it}} \times (1-\Pr_Q(y_{it}=1))^{1-y_{it}} \\ = & \quad \Pi_{i=1}^{L}\Pi_{t=1}^{T}\Phi\left(\beta_t x_i - \alpha_t\right)^{y_{it}} \times (1-\Phi\left(\beta_t x_i - \alpha_t\right))^{1-y_{it}} \end{aligned}$$

NOMINATE Model:

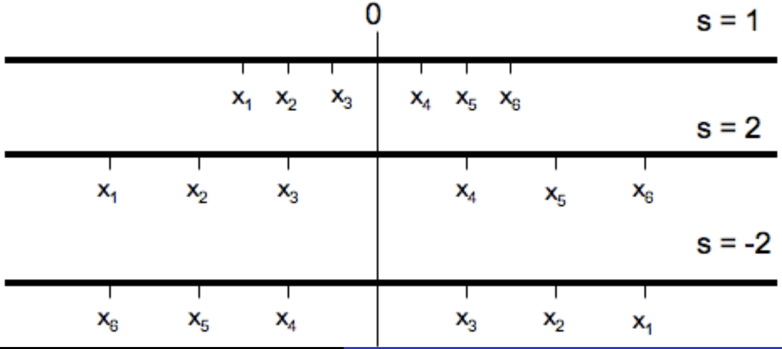
$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) = & \quad \Pi_{i=1}^{L}\Pi_{t=1}^{T}\Pr_N(y_{it}=1)^{y_{it}} \times (1-\Pr_N(y_{it}=1))^{1-y_{it}} \\ = & \quad \Pi_{i=1}^{L}\Pi_{t=1}^{T}\Phi\left[\beta\left[\exp\left(-\frac{1}{2}\left(x_i-\theta_{y(t)}\right)^2\right) - \exp\left(-\frac{1}{2}\left(x_i-\theta_{n(t)}\right)^2\right)\right]\right]^{y_{it}} \\ & \times\left(1-\Phi\left[\beta\left[\exp\left(-\frac{1}{2}\left(x_i-\theta_{y(t)}\right)^2\right) - \exp\left(-\frac{1}{2}\left(x_i-\theta_{n(t)}\right)^2\right)\right]\right]\right)^{1-y_{it}} \end{aligned}$$

Parameter Identification:

Clearly unidentified without further restrictions:

$$x\beta - \alpha = x^*\beta^* - \alpha \text{ with } x^* = xs \text{ and } \beta^* = \beta/s$$
$$\text{Implies } L(\boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{x}|\mathbf{y}) = L(\boldsymbol{\beta}^*, \boldsymbol{\alpha}, \mathbf{x}^*|\mathbf{y})$$

Scale and Rotational Invariance:



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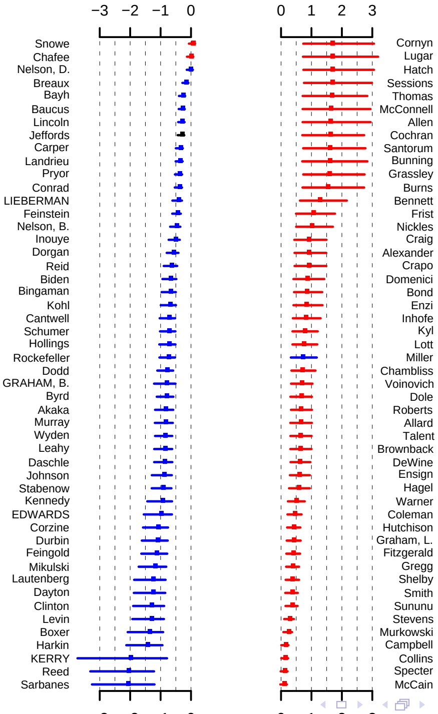
Solution: two linearly independent restrictions on $\mathbf{x}$.

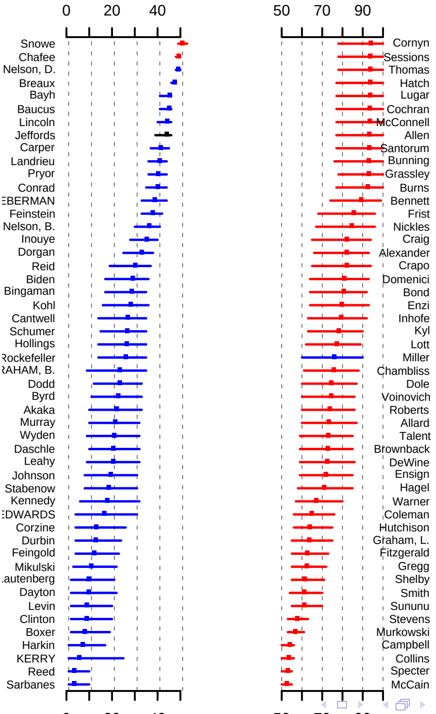
- Sen. Helms = 1, Sen. Kennedy = -1
- $E[\mathbf{x}] = 0$, $V[\mathbf{x}] = 1$, and median Democrat < 0.

Identification Summary

Estimator	**Comparability**	**Constraints**
Interest Group Scores	Not*	None
WNOMINATE	Not	None
DNOMINATE	Across time, not inst. (except pres.)	Parametric Change
DWNOMINATE	Across time, not inst. (except pres.)	Parametric Change
Common Space	Across time and instit.	Fixed $\mathbf{x}$
Quadratic	Depends	Depends



−3 −2 −1 0
0 1 2 3
Snowe
Chafee
Nelson, D.
Breaux
Bayh
Baucus
Lincoln
Jeffords
Carper
Landrieu
Pryor
Conrad
LIEBERMAN
Feinstein
Nelson, B.
Inouye
Dorgan
Reid
Biden
Bingaman
Kohl
Cantwell
Schumer
Hollings
Rockefeller
Dodd
GRAHAM, B.
Byrd
Akaka
Murray
Wyden
Leahy
Daschle
Johnson
Stabenow
Kennedy
EDWARDS
Corzine
Durbin
Feingold
Mikulski
Lautenberg
Dayton
Clinton
Levin
Boxer
Harkin
KERRY
Reed
Sarbanes
Cornyn
Lugar
Hatch
Sessions
Thomas
McConnell
Allen
Cochran
Santorum
Bunning
Grassley
Burns
Bennett
Frist
Nickles
Craig
Alexander
Crapo
Domenici
Bond
Enzi
Inhofe
Kyl
Lott
Miller
Chambliss
Voinovich
Dole
Roberts
Allard
Talent
Brownback
DeWine
Ensign
Hagel
Warner
Coleman
Hutchison
Graham, L.
Fitzgerald
Gregg
Shelby
Smith
Sununu
Stevens
Murkowski
Campbell
Collins
Specter
McCain


0
20
40
50
70
90
Snowe
Chafee
Nelson, D.
Breaux
Bayh
Baucus
Lincoln
Jeffords
Carper
Landrieu
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Computation

Much easier now:

- http://voteview.com/dwnl.htm
- WNOM9707.EXE (WNOMINATE executable)
- WNOMINATE for R
- pscl (ideal) in R
- TurboADA for STATA (see Jeff Lewis, UCLA)
- winBUGS

Recommendations

All produce the same answer (in 1d) for "well-behaved" legislatures.
Slightly prefer quadratic for computational reasons:

- Easier to customize the estimator.
- Measures of parameter uncertainty are recovered directly.
- Code is well-understood (and replicable in other programs).

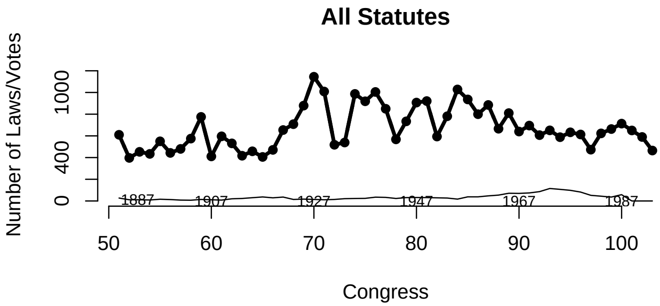
Model Evaluation

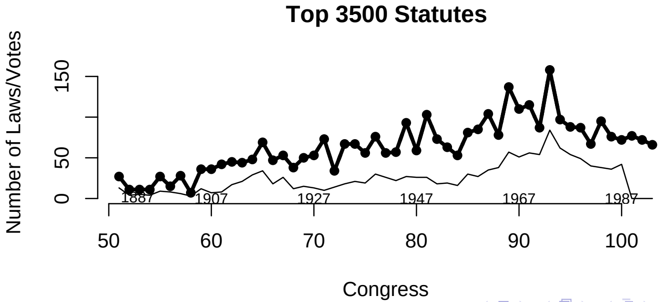
- Error structure.
- Functional form.
- Goodness-of-fit (degrees of freedom).

Exogeneity

How to interpret ideal point estimates:

- Agenda is endogenous & institutional rules change over time.

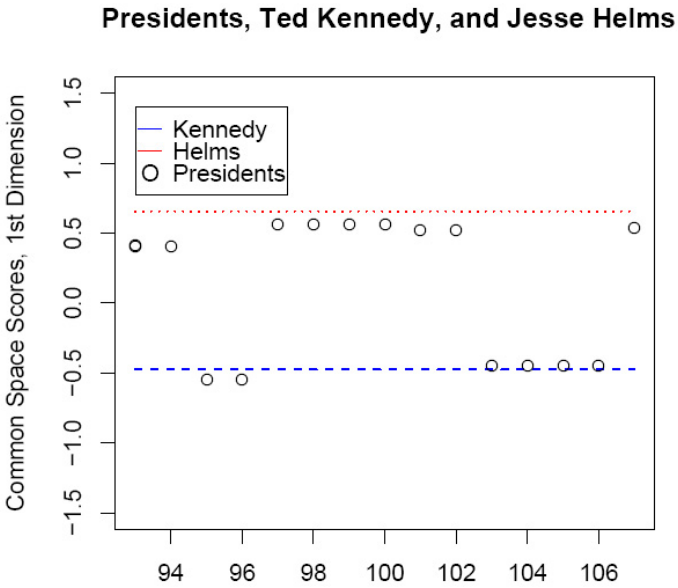

All Statutes
Number of Laws/Votes
0
400
1000
1887
1907
1927
1947
1967
1987
50
60
70
80
90
100
Congress


Top 3500 Statutes
Number of Laws/Votes
0
50
150
1887
1907
1927
1947
1967
1987
50
60
70
80
90
100
Congress

Exogeneity

How to interpret ideal point estimates:

- Agenda is endogenous & institutional rules change over time.
- Decision to vote is endogenous & roll calls not always recorded.


Presidents, Ted Kennedy, and Jesse Helms
Kennedy
Helms
Presidents
Common Space Scores, 1st Dimension
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
94
96
98
100
102
104
106

Exogeneity

How to interpret ideal point estimates:

- Agenda is endogenous & institutional rules change over time.
- Decision to vote is endogenous & roll calls not always recorded.
- Often related to characteristics being explained.

Conclusion

Political science is heavily dependant on measures of latent concepts (e.g., ideal points, party manifestos, expert surveys).

- Computational issues are largely resolved.
- Theoretical and interpretative issues remain.
- Data generating process important (and not well understood).